PRACTICAL SPREADING LAWS: THE SNAKES AND LADDERS OF SHALLOW WATER ACOUSTICS

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Abstract: *Geometrical spreading laws are widely used in underwater acoustics because they provide - if chosen carefully - an accuracy that is sufficient for many applications (source characterisation, impact assessment, sound mapping, regulation) for negligible computation time. The simplest and most widely used form is that corresponding to spherical spreading, with propagation loss,* $PL(R) = 20\log_{10}R$ *dB, which can provide a suitable approximation for deep water. In shallow water, propagation is influenced by multiple reflections from the seabed and sea surface, and a modification is then needed. The resulting effect depends on the kind of source (e.g., continuous or transient) and its directivity (e.g., monopole or dipole). The result often simplifies to the form* $PL(R) = (A + B\log_{10}R)$ *dB, where the values of A and B depend on the conditions. If the source level is known, the received SPL can then be calculated for a continuous source using sound pressure level,* $SPL(R) = SL - PL(R) = SL - (A + B\log_{10}R)$ *dB, but the expected behaviour depends on source directivity, so the values of A and B need to be adjusted accordingly. For a transient source, there are no simple expressions for sound pressure level, but the sound exposure level can be related in a similar way to the energy source level. Guidance is provided for the choice of A and B in shallow water for different activities, including seismic surveys, shipping, explosions, pile driving and use of active sonar. Guidance is also offered on where not to use these simple rules, such as for the calculation of SPL for short transient sources and the pitfalls associated with applying far-field concepts such as source level to distributed sources such as pile drivers.*

Keywords: *Shallow water propagation, Spherical spreading, Cylindrical spreading, Mode stripping, Monopole source, Dipole source, Airgun, Pile driving, Ship, Explosion*

1. INTRODUCTION

Modelling the propagation of sound in shallow water has various applications such as source characterisation, environmental impact assessment, sound mapping and regulation. Given some property of a source such as its source level (SL) in a certain frequency band, what is the sound pressure level (SPL) at some receiver at position **x** relative to the source? This question can be answered if propagation loss (PL) is known because the three quantities are related via

$$\mathrm{SPL}(\mathbf{x}) = \mathrm{SL} - \mathrm{PL}(\mathbf{x}) \tag{1}$$

Propagation loss is a transfer function defined as the difference between SL and SPL. For a monopole source in free space, this transfer function can be written in the form PL = $10\log_{10}\{[W/4\pi I(s)]/r_0^2\}$ dB, where W is the sound power radiated by the source, $I(s)$ is the intensity at distance s from the source and r_0 = 1 m. For the special case of spherical spreading from a monopole source, the intensity $I(s) = W/4\pi s^2$, and PL simplifies in this situation to $10\log_{10}(s/r_0)^2$ dB. Using the upper case symbol R to denote the dimensionless ratio s/r_0, this can also be written PL = $20\log_{10}R$ dB. Sometimes one encounters 'PL = $N\log_{10}R$ dB', often with N = 10, 15 or 20. For N = 20 this expression for PL corresponds to spherical spreading formula, but what does it mean for values of N other than 20?

Some progress can be made towards answering this question with a dimensional argument. The physical quantity represented by any level in decibels is the argument of the $10\log_{10}$ operation before dividing by the reference value. In the case of the transfer function, PL, the physical quantity being expressed in decibels is the reciprocal of the propagation factor [1], which has dimensions of length squared and is equal to the ratio of an area (that area into which the sound has spread at the point it reaches the receiver) to a solid angle (that solid angle from which the sound originates). The reference value of this physical quantity is r_0^2 = 1 m^2. Considering the expression PL = $(A+B\log_{10}R)$ dB, the above dimensional argument has implications for the relationship between A and B. Basically, if A is the level of some physical quantity a, the value of B determines the dimensions of a. If B = 20, a is dimensionless. More generally, using the notation [L] to indicate a physical quantity with dimensions of length, the dimensions of a are $[\mathrm{L}]^{2-B/10}$. Examples are cylindrical spreading, for which B = 10 and $A = 10\log_{10}([\mathrm{L}]/\mathrm{r0})$, and mode stripping, for which B =15 and $A = 10\log_{10}([\mathrm{L}]/r_0)^{1/2}$. In both cases the distance represented by [L] is proportional to the water depth. Section 2 describes basic properties of underwater sound for a continuous monopole source. Effects of proximity to the sea surface and of transients are described in Secs. 3 and 4, respectively. Distributed sources are considered in Sec. 5. A range independent environment is assumed throughout, with uniform sound speed and negligible absorption in the water. In reality, deviations from these assumptions will result in deviation from 'practical' geometrical spreading.

2. BASIC PROPERTIES OF UNDERWATER SOUND

The quantities SPL, SL and PL are introduced in turn in this section. SPL is defined in terms of the RMS sound pressure, p_{RMS}, as [2]

$$\mathrm{SPL} \equiv 10\log_{10}\frac{p_{\mathrm{RMS}}^{2}}{p_{0}^{2}}\ \mathrm{dB} \tag{3}$$

The reference sound pressure is $p_0 = 1$ μPa, the physical quantity expressed here as a level is mean square sound pressure p_{RMS}^2 and the corresponding reference value is p_0^2.

SL is a property of the source closely that is closely related to its radiated sound power W. For a monopole in free space, of uniform density ρ and sound speed c, these two quantities are related via SL = $10\log_{10}[(\rho c/4\pi)\ W/(p_0^2\ r_0^2)]$ dB. More generally, SL can be defined in terms of the source factor S [1]

$$\mathrm{SL} \equiv 10\log_{10}\frac{S}{p_{0}^{2}r_{0}^{2}}\ \mathrm{dB} \tag{4}$$

where $S = p_{\mathrm{FF}}(s)^2\ s^2$, and $p_{\mathrm{FF}}(s)$ is the RMS sound pressure at distance s from the source and in its far field (i.e., unaffected by reflections from boundaries). The source factor is not a property of the sound field but of the source. The far-field sound pressure is that sound pressure that *would* exist in the far field of that source if it were placed in an unbounded loss-free medium with the same density and sound speed as the true medium at the source position, and excited with identical motion on all acoustically active surfaces as in the true medium.

In Eq. (4), the physical quantity expressed as a level is the source factor S. That quantity has dimensions (pressure times distance) squared, and its reference value is $p_0^2\ r_0^2 = 1$ μPa^2 m^2. Propagation loss is defined as [1] $\mathrm{PL} \equiv \mathrm{SL} - \mathrm{SPL}$. Equation (1) follows from this definition. Further, substituting Eqs. (3), (4) in Eq. (1), it follows that

$$\mathrm{PL} = 10\log_{10}\frac{S/p_{\mathrm{RMS}}^{2}}{r_{0}^{2}}\ \mathrm{dB}\,. \tag{5}$$

At a small distance s from the source, the area into which the sound has spread is $4\pi s^2$. The solid angle at the source ensonifying this area is 4π, so the reciprocal of the propagation factor is $4\pi s^2/4\pi = s^2$, corresponding to spherical spreading ($B = 20$; $A = 0$):

$$\mathrm{PL} = 10\log_{10}\frac{s^{2}}{r_{0}^{2}}\ \mathrm{dB}\,. \tag{6}$$

Further away (several water depths) from the source the area becomes $2\pi rH$, where r is the horizontal distance and H is the water depth. For a seabed critical angle ψ, the vertical range of angles that propagates in the cylindrical spreading region is $-\psi$ to $+\psi$, corresponding to a vertical opening angle of 2ψ. The range of azimuth angles is 2π. The solid angle is therefore 2ψ multiplied by 2π, i.e., $4\pi\psi$, leading to the expression for cylindrical spreading in shallow water at frequencies above the shallow water cut-off frequency: [1, 3]

$$\mathrm{PL} = 10\log_{10}\frac{r}{r_0}\,\mathrm{dB} + 10\log_{10}\frac{H/(2\psi)}{r_0}\,\mathrm{dB}\,, \tag{7}$$

and corresponding to B = 10 and A = $10\log_{10}[H/(2\psi r_0)]$. In Eq. (7) and subsequent equations in this section, r is the horizontal range (not the slant range). The change in distance variable from s to r, made for convenience, makes little material difference at the longer distances of relevance here.

After multiple bottom reflections the energy in steep paths close to ψ is dissipated more rapidly than paths close to horizontal, reducing the effective propagation angle to [1] $\theta_{\mathrm{eff}} = (\pi H/4\eta r)^{1/2}$. Replacing ψ with θ_{eff} in Eq. (7) (the area is unchanged) gives the usual "15logR" mode stripping formula: [1, 3]

$$\mathrm{PL} = 10\log_{10}\frac{r^{3/2}}{r_0^{3/2}}\,\mathrm{dB} + 10\log_{10}\frac{(\eta H/\pi)^{1/2}}{r_0^{1/2}}\,\mathrm{dB}\,, \tag{8}$$

corresponding to B = 15 and A = $5\log_{10}[\eta H/(\pi r_0)]$, implying that $A \neq 0$ unless $\eta H = \pi r_0$. If A is arbitrarily set to zero in Eq. (2), for a silt or sand seabed, the magnitude of the resulting error is typically up to about 5 dB in very shallow water ($H \sim 1$ m), increasing to 15-20 dB for deeper water ($H \sim 100$ m).

Figure 1 (left) shows PL(r) calculated using Equations (6) to (8). These equations may be used with Eq. (1) to predict SPL for a continuous source, such as a sonar projector, deployed sufficiently far from all boundaries that its radiation impedance is unaffected by the presence of any boundary. Effects of the sea surface and of transient sound are discussed in Secs. 3 and 4, respectively.

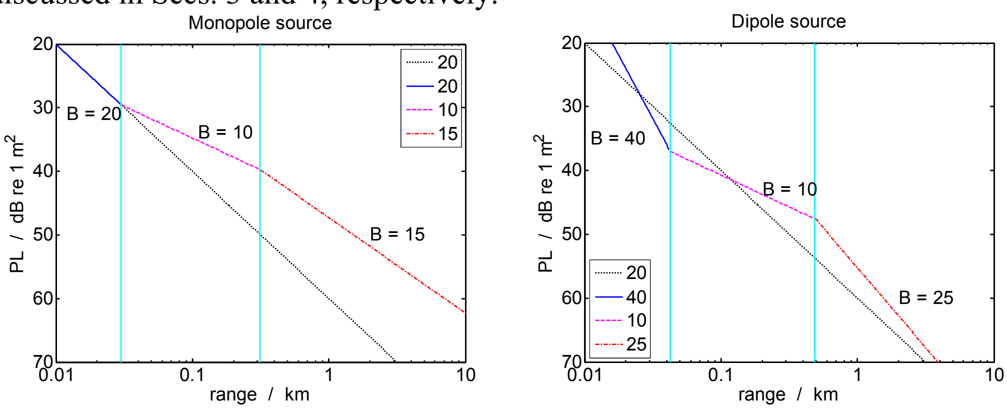

Fig.1: Propagation loss vs range. left: monopole (H = 30 m, ψ = 0.5 rad, η = 0.3 Np/rad); right: dipole (frequency = 300 Hz, z = 1 m; D = 10 m; other parameters as for monopole). Vertical cyan lines separate regimes of spherical spreading (left), cylindrical spreading (middle) and mode stripping (right).

3. PROXIMITY TO THE SEA SURFACE

The derivation in Sec. 2 assumes a monopole source not close to a boundary (in terms of acoustic wavelength), and especially not close to the sea surface. Departures from this assumption are considered next. The sound field in the vicinity of a point source in close proximity to the sea surface is strongly influenced by the reflected field, which experiences a π phase change, resulting in a Lloyd mirror interference pattern. In this situation, Eq. (1) still holds in the form SPL = MSL – PL , where MSL is the monopole source level [1]. The effect of the sea surface is to modify the received field for a given source level (because of interference between the direct and surface reflected paths) in such a way that Eqs. (6)-(8) no longer hold in general. In this situation the dependence of PL on range depends on frequency. The effect of this frequency dependence on the values of A and B is considered next.

For spherical spreading, it is useful to define a coordinate system with its origin at the centre of the dipole, on the sea surface directly above the monopole (at depth z). If s is the distance from the origin in this co-ordinate system, and θ the elevation angle such that for receiver depth D, $\theta = \arcsin(D/s)$, the spherical spreading PL can be written [1]

$$\mathrm{PL} = 10\log_{10}\frac{s^2}{r_0^{\ 2}}\ \mathrm{dB} - 10\log_{10}\left[4\sin^2(kz\sin\theta)\right]\mathrm{dB} \tag{9}$$

where k is the acoustic wavenumber in water. This can be written in the form of Eq. (2), with B = 20 and $A = -10\log_{10}[4\sin^2(kz\sin\theta)]$, although the value of A is only a constant if the elevation angle θ is also a constant. If instead of keeping the angle fixed it is the depth D that is kept fixed, in the long range (small argument) limit the values of the constants become $B \approx 40$ and $A \approx -10\log_{10}[4(kzD/r_0)^2]$.

In the cylindrical spreading region the applicable power law becomes [4]

$$\mathrm{PL} = 10\log_{10}\frac{r}{r_0}\ \mathrm{dB} + 10\log_{10}\frac{3H/\left(k^2z^2\psi^3\right)}{r_0}\ \mathrm{dB} \tag{10}$$

(assuming $kz < \pi/4$, here and for the remainder of Sec. 3), corresponding to B = 10 and $A = 10\log_{10}\left[3H/\left(k^2z^2\psi^3 r_0\right)\right]$. The corresponding equation in the mode stripping region is [4]

$$\mathrm{PL} = 10\log_{10}\frac{r^{5/2}}{r_0^{\ 5/2}}\ \mathrm{dB} + 10\log_{10}\frac{4(kz)^{-2}\left(\eta^3/\pi H\right)^{1/2}}{r_0^{\ -1/2}}\ \mathrm{dB}, \tag{11}$$

corresponding to B = 25 and $A = 10\log_{10}\left[4(kz)^{-2}\left(\eta^3 r_0/\pi H\right)^{1/2}\right]$. Equations (9) to (11) are plotted in the right hand graph of Fig. 1.

All of the above assumes that SL is the monopole source level, in other words it is the source level of a sound source in isolation, without the contribution of (e.g.) the surface image if the source is close to surface. Sometimes cited for such a source is the source level of the dipole formed by itself plus surface-reflected image combined. Such a dipole source level ($\mathrm{SL_{dp}}$) is not suitable for use in Eq. (1). [5]

Current measurement standards for underwater radiated noise of surface ships do not take into account the actual propagation loss at the test location. The reported quantity is a so-called radiated noise level (RNL), which represents the measured SPL, scaled by the distance between ship and hydrophone under the assumption of spherical spreading (Eq. 6). [5] This RNL is not suitable for use in Eq. (1). Given sufficient information about the measurement geometry and environmental parameters, the propagation loss could be estimated for a monopole below the sea surface. This concept provides an acceptable approximation for noise radiation that is due to propeller cavitation, but is less accurate for other source mechanisms, like machinery noise, which is radiated from the ship hull as a distributed source in the proximity of the water surface – see also Sec. 5.

4. TRANSIENT SOURCES

All equations presented up to this point apply to a steady state situation, implying that the duration of the sound is sufficiently large for the steady state to be reached. A different approach is needed for sounds that do not reach a steady state. In that situation it is more useful to cast equations in terms of time-integrated quantities such as sound exposure instead of time-averaged ones such as mean-square sound pressure. More specifically, the simplicity and functional form of Eq. (1) is retained for a transient, with the same expressions for PL(r), if SPL is replaced with sound exposure level (SEL) and SL with the energy source level (ESL). With these same substitutions in Fig. 1 it follows that those graphs (monopole or dipole version as appropriate, depending on acoustic frequency and proximity of source to the sea surface) apply also to transient sources such as explosions (if far enough away to avoid the non-linear shock wave) and airguns.

Seismic airgun sources typically comprise several tens of airguns in an array. Each airgun releases a bubble of compressed air when commanded to do so. If all the airguns are at the same depth they are usually fired simultaneously.

For the purposes of imaging, the airgun array is usually represented as an array of monopoles (one for each bubble) and each monopole is characterised by a source function called the "notional source signature". The notional source signatures can be determined using near-field hydrophones [6] or they can be modelled [7, 8] or the they can be approximated by notional source signatures measured for individual air-guns [9].

It is usual to characterise arrays of airguns by their time-domain source signature, defined as the far-field product of sound pressure and distance: $S_{\mathrm{dp}}(t) = p(r,t)\, r$, a property that is independent of distance from the source, where the distance r is usually taken to be in a direction vertically beneath the airgun array. This quantity is related to dipole source energy level ($\mathrm{ESL_{dp}}$), by implication also in the vertical direction, by the formula

$$\mathrm{ESL_{dp}} \equiv 10 \log_{10} \frac{\int S_{\mathrm{dp}}(t)^2 \,\mathrm{d}t}{p_0^{\,2} r_0^{\,2} t_0^{\,2}} \ \mathrm{dB} \tag{12}$$

As with RNL and $\mathrm{SL_{dp}}$ mentioned previously, this quantity needs to be converted to a monopole source level before it can be used in Eq. (1).

5. DISTRIBUTED SOURCES

Up to this point the concepts of PL and SL are used to characterise the received field via SPL = SL – PL or SEL = ESL – PL. Both PL and SL are defined in terms of the far field of the source. In this section we consider sources that span the entire water column. Such distributed sources are less well understood than the sources considered previously. We have in mind pile-driving equipment typically used for offshore construction in shallow water. Because of the difficulty in identifying the far field of such sources, the concepts of SL and PL then need generalisation before they become applicable, with or without simple spreading laws.[10] For the case of impact pile driving, Reinhall and Dahl [11] invoke a complex-phased array of point sources of fixed strength and linearly varying phase that depends on depth of the point source in order to predict the radiated sound field as function of range from the pile. The phasing establishes angular and depth dependence in the pressure field that is verified with field observation. [11, 12]

An important range scale emerges (Fig. 2a) from this approach [13] equal to r^* = depth/tanθ, where angle θ is related to the ensuing Mach wave; a typical $\theta \sim 17^o$ puts r^* at about 3 times the water depth. For ranges $r/r^* < 1$ the field strength depends strongly on measurement depth, D, whereas for $r/r^* > 1$ this is less so, with significant implications on the allocation of measurement effort for environmental noise monitoring for bridge or wind farm construction. A depth average of the field from impact pile driving (Fig. 2b) is consistent with cylindrical (not spherical) spreading at least for ranges up to r^*, as expected from energy-conservation [14]. For a situation involving uniform water depth, consistent with the assumption of a range-independent environment made throughout, the reduction of SEL with range is expected to go as SEL = constant $-$ (10log$_{10}R$ dB + αr), where the exponential decay arises from multiple reflections from the seabed. The constant in this expression is a logarithmic measure of the sound energy radiated as a consequence of the hammer impact. It is not source level and does not have the dimensions of source level.

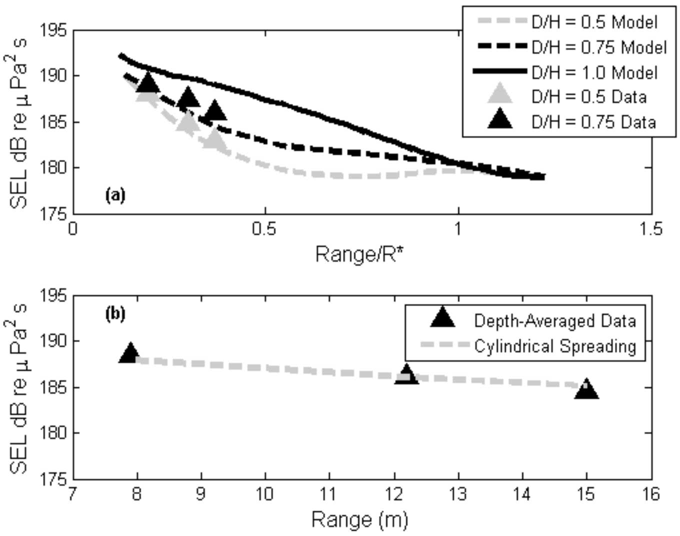

Fig.2: a) Measurements (symbols) of SEL as function of scaled range r/r for two hydrophone depths compared with PE simulation (lines), where results in both cases are plotted as a function of range is scaled by r*. b) Depth-averaged measurements of SEL at three ranges (averaging depth 4.9-10.5 m) compared with equivalent averages of PE simulated data made at 1 m range intervals. The dashed line represents cylindrical spreading. For details, see Ref. [13].*

6. CONCLUSIONS

The basic equations for propagation loss look (and are) simple: Eq. (1) and related equations for SPL, SEL; Eqs. (6) to (13) for PL, but there are some rules. If propagation loss is written in the form ($A + B$ log$_{10}R$) dB, the values of A and B are not arbitrary and not independent of one another. Special cases considered include a surface ship, airgun and pile driving. In all cases except pile driving there is a region ("regime") of spherical spreading followed by cylindrical spreading and then mode stripping. The "regime" determines the value of *B*, which in turn determines the functional form (dependence on frequency, water depth, etc) of *A*.

Possible numerical values for *B* include 10, 15, 20, 25 and 40. The "constant" *A* can be equal to 0 dB, but in most situations takes some other value (see especially Secs. 2 and 3). Pile driving is a special case that is less well understood than the other sources considered, and for which a generalisation of the concept of source level and propagation loss is the subject of ongoing research.

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